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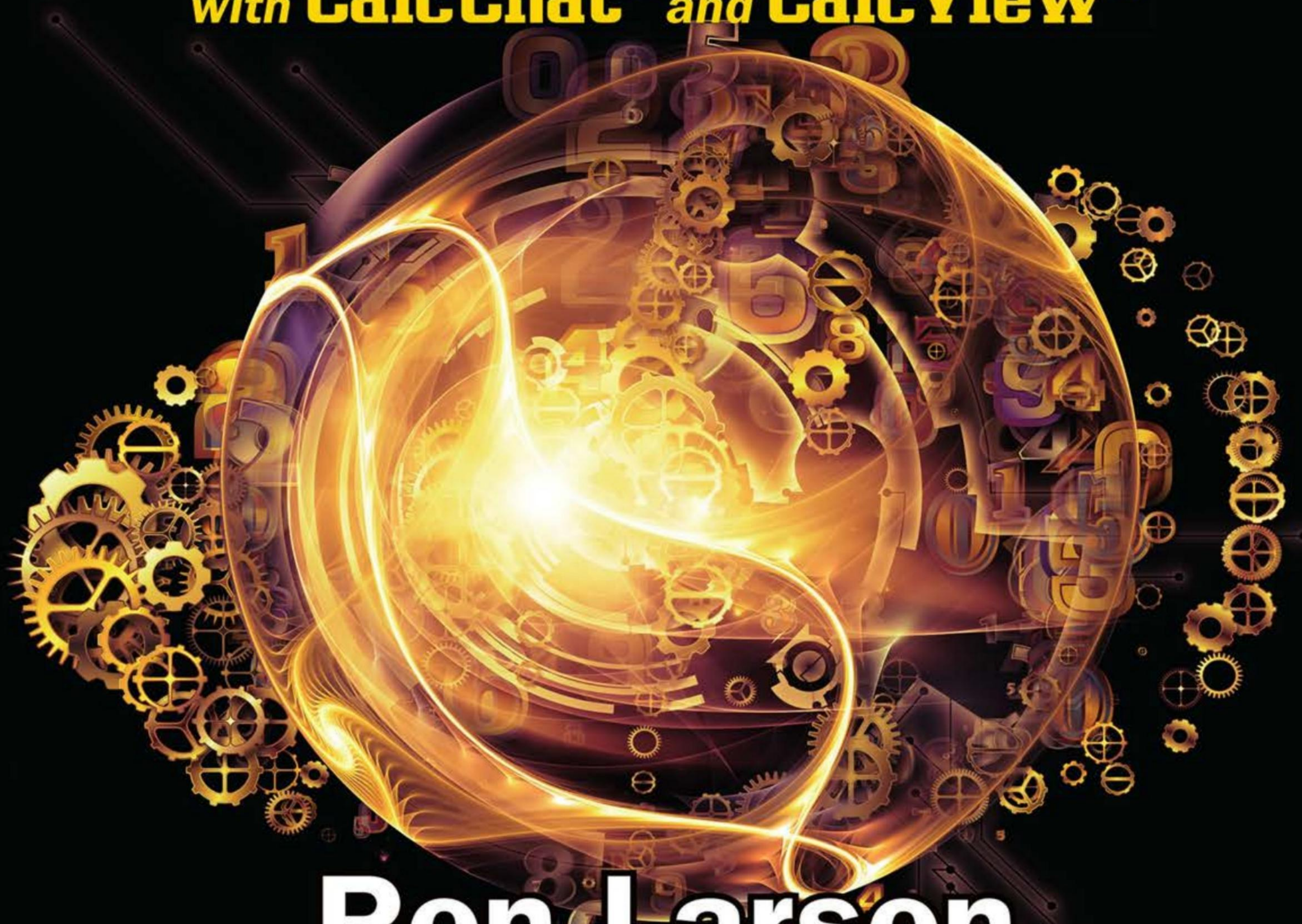

Ron Larson

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DERIVATIVES AND INTEGRALS

Basic Differentiation Rules

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'), \ u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$
12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$
14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$
20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$
21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$
22. $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1 + u^2}$
23. $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$
24. $\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$

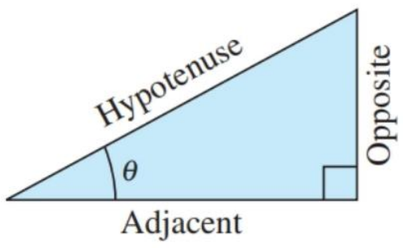
Basic Integration Formulas

1. $\int kf(u)\,du = k\int f(u)\,du$
2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$
4. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \ n \neq -1$
5. $\int \frac{du}{u} = \ln|u| + C$
6. $\int e^u\,du = e^u + C$
7. $\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$
8. $\int \sin u\,du = -\cos u + C$
9. $\int \cos u\,du = \sin u + C$
10. $\int \tan u\,du = -\ln|\cos u| + C$
11. $\int \cot u\,du = \ln|\sin u| + C$
12. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
13. $\int \csc u\,du = -\ln|\csc u + \cot u| + C$
14. $\int \sec^2 u\,du = \tan u + C$
15. $\int \csc^2 u\,du = -\cot u + C$
16. $\int \sec u \tan u\,du = \sec u + C$
17. $\int \csc u \cot u\,du = -\csc u + C$
18. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C$
19. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$
20. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

TRIGONOMETRY

Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

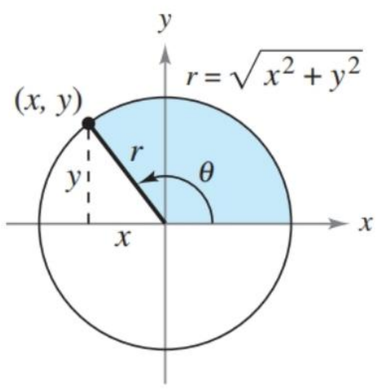


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \quad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \quad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

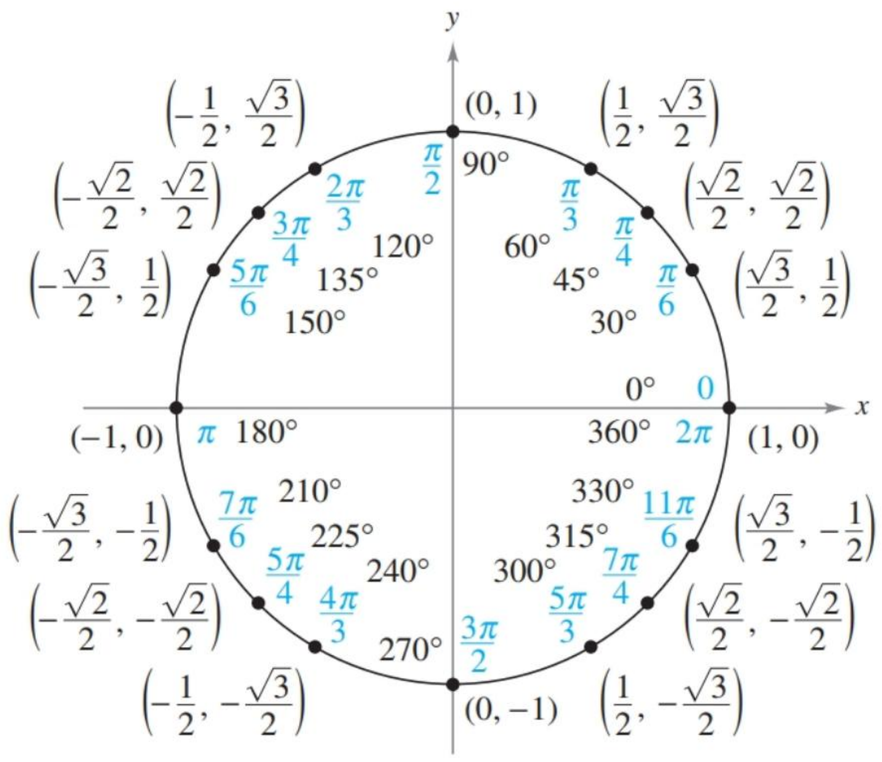
Circular function definitions, where θ *is any angle.*



$$\sin\theta = \frac{y}{r} \quad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \quad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \quad \cot\theta = \frac{x}{y}$$



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \sec x = \frac{1}{\cos x} \quad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \quad \cos x = \frac{1}{\sec x} \quad \cot x = \frac{1}{\tan x}$$

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \quad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Even/Odd Identities

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x$$

$$\csc(-x) = -\csc x \quad \tan(-x) = -\tan x$$

$$\sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

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The Pennsylvania State University
The Behrend College

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Brentwood Academy

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with CalcChat® and CalcView®
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Preface

Welcome to *Calculus for AP*® Second Edition. We are excited to offer you a new edition with even more resources that completely support the concepts and goals presented in the course frameworks for AP® Calculus AB and AP® Calculus BC.

Our main goals for this textbook are to provide you with the tools you need to understand calculus concepts and prepare you for the AP® Calculus Exams. We think you will find that this text, along with its resources, will help you to achieve these goals. Additionally, we are pleased and excited to offer you the companion websites listed below.

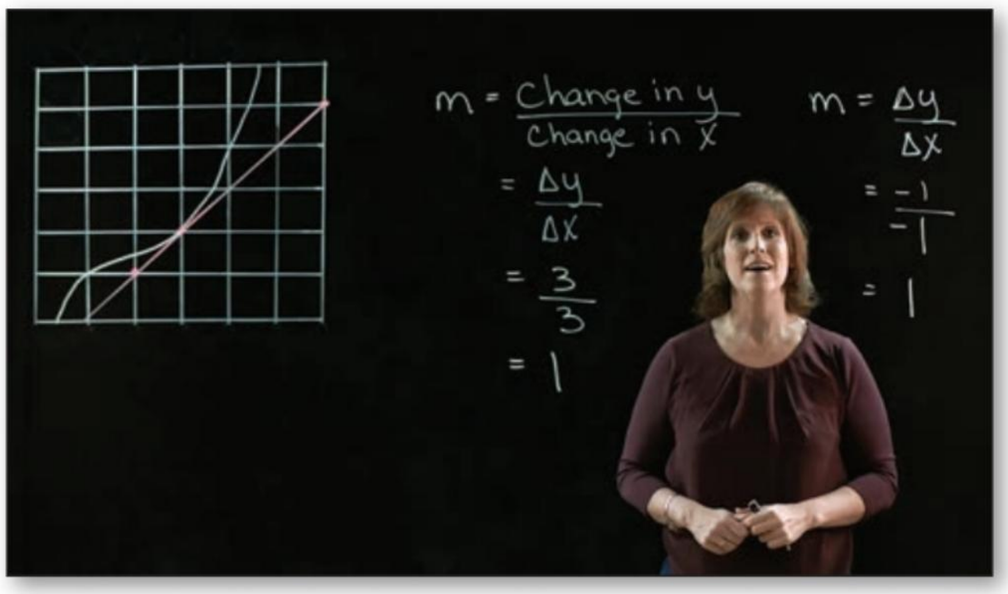
- **LarsonCalculusforAP.com —** companion website with resources to supplement your learning
- **CalcView.com —** video solutions to selected exercises
- **CalcChat.com —** worked-out solutions to odd-numbered exercises and access to online tutors

These websites will help enhance and reinforce your understanding of the material presented in this text and help you master calculus. Also, these websites are **free** to access, and you do **not** have to create an account to use them.

Features

UPDATED LarsonCalculusforAP.com

All website features have been updated based on the revisions to the text. Watch videos explaining mathematical concepts or proofs, explore examples, view three-dimensional graphs, review sample scoring for free-response questions, and much more.



Video solutions of selected exercises are at CalcView.com.

UPDATED CalcView®

The website CalcView.com contains video solutions of selected exercises. Watch instructors progress step-by-step through solutions, providing guidance to help you solve the exercises. The CalcView mobile app is available for free at the Apple® App Store® or Google Play™ store. The app features an embedded QR Code® reader that can be used to scan the on-page codes and go directly to the videos. You can also access the videos at CalcView.com.

UPDATED CalcChat®

In the section and review exercises, be sure to notice the reference to CalcChat.com. This website provides free step-by-step solutions to all odd-numbered section and review exercises. Also, you can chat with a tutor, at no charge, during the hours posted at the site. Over the years, millions of students have visited this site for help. The CalcChat mobile app is available for free at the Apple® App Store® or Google Play™ store.

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*Available at the text-specific website LarsonCalculusforAP.com

REVISED Side-By-Side Examples

Throughout the text, we present solutions to examples from multiple perspectives— algebraically, graphically, and numerically. The side-by-side format of this pedagogical feature helps you to see that a problem can be solved in more than one way and to see that different methods yield the same result. The side-by-side format also addresses many different learning styles. Many of these examples have been revised for clarity.

REVISED Algebra Reviews

Algebra Review notes appear throughout each chapter and offer algebraic support at point of use. This support is revisited in a two-page algebra review for each chapter in Appendix D, where additional details of example solutions with explanations are provided. Per user feedback, these examples have been revised in this edition to provide more algebraic help.

REVISED Insights

Throughout the book, *Insights* offer important information to help you prepare for the AP® Exam. These notes have been updated to reflect the changes in the new course frameworks for AP® Calculus AB and AP® Calculus BC.

Insight

On the AP® Exam, a Riemann sum is called a *left Riemann sum* when c_i is the left endpoint of $[x_{i-1}, x_i]$, a *right Riemann sum* when c_i is the right endpoint of $[x_{i-1}, x_i]$, and a *midpoint Riemann sum* when c_i is the midpoint of $[x_{i-1}, x_i]$.

REVISED Applications

Examples and exercises are included throughout the text to address the question, "When will I use this?" These applications are pulled from diverse sources, such as current events, world data, industry trends, and more, and relate to a wide range of interests. Understanding where calculus is used promotes better understanding of the material. We have updated the applications to use more current data and to include suggestions from our users.

REVISED Exercise Sets

The exercise sets have been carefully and extensively examined to ensure they are rigorous and relevant and to include topics our users have suggested. The exercises are organized and titled so you can better see the connection between examples and exercises, as well as the connection between exercises and the AP® Calculus AB and BC mathematical practices. (See page xi.) Multi-step, real-life exercises reinforce problem-solving skills and mastery of concepts by giving you the opportunity to apply the concepts in real-life situations. Per user feedback, we have added more particle motion problems.

REVISED Calculus AP® – Exam Preparation Questions

These questions appear in each section in Chapters 1–9 and are modeled after the types of questions you will encounter on the AP® Exam. These include multiple-choice and free-response questions. These questions have been updated to reflect the changes in the new course frameworks for AP® Calculus AB and AP® Calculus BC.

REVISED Section Projects

Projects appear in selected sections and encourage you to explore applications related to the topics you are studying. We have added new projects, revised others, and kept some of our favorites. All of these projects provide an interesting and engaging way for you and other students to work and investigate ideas collaboratively.

REVISED What You Need to Know

The *What You Need to Know* feature accompanies each set of AP® Exam Practice Questions appearing at the end of Chapters 1–9. This feature provides tips to help you prepare for the AP® Exam. These tips have been updated to reflect the changes in the new course frameworks for AP® Calculus AB and AP® Calculus BC.

REVISED AP® Exam Practice Questions

These questions appear at the end of Chapters 1–9 and are modeled after the types of questions you will encounter on the AP® Exam. These include multiple-choice and free-response questions. These questions have been updated to reflect the changes in the new course frameworks for AP® Calculus AB and AP® Calculus BC.

Section Objectives

At the beginning of each section, there is a list of learning objectives that provides you with the opportunity to preview what will be presented in the upcoming section.

Theorems

Theorems provide the conceptual framework for calculus. Theorems are clearly stated and separated from the rest of the text by boxes for quick visual reference. Key proofs often follow the theorem and can be found at LarsonCalculusforAP.com.

Proof Videos

Watch videos that explain the proofs of theorems in the text at LarsonCalculusforAP.com.

Definitions

As with theorems, definitions are clearly stated using precise, formal wording and are separated from the text by boxes for quick visual reference.

Remarks

These hints and tips reinforce or expand upon concepts, help you learn how to study mathematics, caution you about common errors, address special cases, or show alternative or additional steps to a solution of an example.

Explorations

Explorations provide unique challenges to study concepts that have not yet been formally covered in the text. They allow you to learn by discovery and introduce topics related to ones presently being studied. Exploring topics in this way encourages you to think outside the box.

Interactive Examples

Examples throughout the book are accompanied by *Interactive Examples* at LarsonCalculusforAP.com. These interactive examples allow you to explore calculus by manipulating functions or graphs and observing the results.

Technology and Technology Pitfall Notes

Throughout the text, *Technology* notes show you how to use technology to solve problems and explore concepts of calculus. *Technology Pitfall* notes point out some hidden difficulties of using technology.

How Do You See It?

The *How Do You See It*? exercise in each section presents a problem that you will solve by visual inspection using the concepts learned in the lesson. This exercise is excellent for classroom discussion.

Performance Tasks

Performance Tasks appear at the end of each chapter and ask you to demonstrate your knowledge and understanding of material in the chapter.

AP® Calculus AB and BC Mathematical Practices

AP® Calculus AB and BC mathematical practices are embedded in the study of calculus and are intended to help students establish lines of reasoning that will enable them to apply calculus concepts to solve problems.

Mathematical Practice 1: Implementing Mathematical Processes

Determine expressions and values using mathematical procedures and rules.

- Reinforced throughout the text with *Implementing Processes* notes (see example at the right) and exercises.
- Side-by-side examples throughout this text demonstrate how to solve problems using different mathematical procedures.

Implementing Processes

Before differentiating functions involving radicals, rewrite the function with rational exponents.

Mathematical Practice 2: Connecting Representations

Translate mathematical information from a single representation or across multiple representations.

- Reinforced throughout the text with *Connecting Representations* notes and exercises (see example at the right).
- Students are provided ample opportunities to solve problems graphically, numerically, and analytically in the exercises as well as in the *Explorations*.
- Students identify mathematical information from graphical, numerical, analytical, or verbal representations, such as writing a definite integral given a graph.

105. Connecting Representations Give a geometric explanation of why

$$\int_0^{\pi/2} x \sin x \, dx \leq \int_0^{\pi/2} x \, dx.$$

Verify the inequality by evaluating the integrals.

Mathematical Practice 3: Justification

Justify reasoning and solutions.

- Reinforced throughout the text with *Justification* notes and *Justifying* exercises (see example at the right).
- Students can confirm that solutions are accurate and appropriate. For example, students should verify answers to integration problems by differentiating.

Mathematical Practice 4: Communication and Notation

Use correct notation, language, and mathematical conventions to communicate results or solutions.

- Reinforced throughout the text with *Communication and Notation* notes (see example at the left) and exercises.
- A variety of notations are used throughout the book so students become familiar with the different ways a concept can be presented.
- The connections between notations and definitions are explained throughout the text.
- Students are continually asked to interpret their results, explain their reasoning, and justify their answers in the problem-solving process.
- *Exploring Concepts, Writing, Think About It, Investigation, How Do You See It?, Error Analysis,* and *Proof* exercises require students to use clear, precise mathematical language in their solutions and explanations.

Communication and Notation

The unit for the area of a region defined by a rate of change is the unit for the rate of change multiplied by the unit for the independent variable. For instance, in Example 2, the unit for $v(t)$ is feet per second and the unit for t is seconds, so the unit for the area is

$$\frac{\text{feet}}{\cancel{\text{second}}} \cdot \cancel{\text{second}} = \text{feet}.$$

Go to www.collegeboard.org for more information about the AP® Calculus AB and BC mathematical practices.

Student Resources

Fast Track to a 5 Workbook (978-0-357-52033-8)
This AP® test preparation manual provides valuable test-taking strategies, review topics based on the course frameworks for AP® Calculus AB and AP® Calculus BC, and full-length diagnostic and practice exams. Keyed to this text, it helps students efficiently and effectively prepare for the AP® exam.

Student Solutions Manual (978-0-357-52034-5)
This guide offers step-by-step solutions for all odd-numbered exercises.

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Instant Access Code: 978-0-357-52043-7
WebAssign® combines exceptional *Calculus for AP®* content with powerful online homework and assessment solutions. WebAssign® engages you with assignable learning resources, tutorial content, and an interactive eBook—MindTap, helping you to develop a deeper conceptual understanding of the subject matter.

LarsonCalculusforAP.com
Of the many features at this website, students have told us that the videos are the most helpful. Watch instructional videos presented by Dana Mosely, as he explains various calculus concepts. Watch proof videos presented by Bruce Edwards, as he explains various calculus theorems and their proofs. Use a QR Code® reader to scan the on-page codes near the theorems and go directly to the videos. Two other helpful features are the data downloads (editable spreadsheets so you do not have to enter the data) and the AP® Prep Solutions for the end-of-chapter AP® Exam Practice Questions.

CalcChat.com
This website provides free step-by-step solutions to all odd-numbered section and review exercises. Additionally, you can chat with a tutor, at no charge, during the hours posted at the site.

CalcView.com
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Teacher Resources

Teacher's Edition (978-0-357-52031-4)
The Teacher's Edition for *Calculus for AP®* is the complete student text that includes annotations for teachers, as well as wrap-around margins. In these margins, teachers will find notes from co-author Paul Battaglia—Chapter Planning Guides, Section Overviews, Essential Questions, Lesson Motivators, Teaching Strategies, Extra Examples, Common Errors, Mathematical Practice notes, Lesson Closers, Assignment Guides, Activities, Sample Grading Rubrics to the Performance Tasks, and answers to all exercises in the student edition. The Teacher's Edition also includes a correlation to the AP® Calculus AB and AP® Calculus BC curriculum framework and a pacing guide.

Complete Solutions Manual (978-0-357-52035-2)
This manual contains solutions to all exercises in the text. This can be found on the instructor companion site.

Teacher's Resource Guide (978-0-357-52032-1)
This robust manual contains an abundance of resources keyed to the *Calculus for AP®* text at the section and chapter level, including section objectives, teaching tips, and chapter projects.

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WebAssign® combines exceptional *Calculus for AP®* content with the most powerful online homework solution. WebAssign® engages your students with assignable learning resources, tutorial content, and an interactive eBook—MindTap, helping you to develop a deeper conceptual understanding of the subject matter.

Cengage Learning Testing Powered by Cognero
This flexible online system offers additional AP® practice questions; allows you to author, edit, and manage test bank content; create multiple test versions in an instant; and deliver tests from your LMS, your classroom, or wherever you want. This is available online via *login.cengage.com*.

Instructor Companion Site
Everything you need for your course in one place! This collection of book-specific lecture and class tools is available online via *login.cengage.com*. Access and download PowerPoint presentations, the Solutions Manual, the Teacher's Resource Guide, and more.

LarsonCalculusforAP.com
In addition to its student resources, LarsonCalculusforAP.com also has resources to help teachers. Watch co-author Paul Battaglia discuss the key points and concepts of each section and provide teaching tips and strategies. Use the biographical sketches to show your students the history of calculus and the mathematicians who developed it.

MathGraphs.com
For exercises that ask students to draw on the graph, we have provided **free**, printable graphs at MathGraphs.com.

MathArticles.com
MathArticles.com provides you and your students with over 40 relevant articles from renowned math journals.

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If you have suggestions for improving this text, please feel free to write to us. Over the years we have received many useful comments from both teachers and students, and we value these very much.

Ron Larson
Paul Battaglia

Preparing for the AP® Calculus Examination

Taking an AP® course can be exhilarating. Whether you are taking an AP® course at your school or you are working on the AP® curriculum independently, the stage is set for a great intellectual experience.

But sometime in the spring, when the examination begins to loom on a very real horizon, your AP® course can seem intimidating. It is a normal feeling to be nervous about the test; you are in good company.

The best way to deal with an AP® examination is to master it, not let it master you. You should think of this examination as a way to show off how much calculus you know. Attitude *does* help. But, no matter what type of math student you are, there is still a lot you can do to prepare for the exam. Focused review and practice time will help you master the examination so that you can walk in with confidence and earn a great score.

What's in *Calculus for AP®* that will help you prepare

As you work through the textbook, there are some things that you should do to get the most out of the text.

- As you read the sections,
  - read the notes about the AP® Calculus AB and BC mathematical practices: Implementing Processes, Connecting Representations, Justification, and Communication and Notation,
  - read the Remarks,
  - do the Explorations,
  - read the notes on Technology, and
  - take time to read the examples, especially the ones with multiple parts that try to represent various ways to look at similar problems and notice how different their solution methods might be.
- After finishing a section or the chapter, work through
  - the exercises about the AP® Calculus AB and BC mathematical practices: Implementing Processes, Connecting Representations, Justification, and Communication and Notation,
  - the True or False exercises to help you discern small differences in conceptual understanding,
  - the Think About It exercises and the Exploring Concepts exercises to help you improve your conceptual understanding and your ability to explain your thinking,
  - the How Do You See It? exercises to process the information—to help you compare methods and/or look at concepts,
  - the Review Exercises,
  - the Calculus AP®—Exam Preparation Questions,
  - the AP® Exam Practice Questions to practice your test-taking skills and understanding of the material you just studied, and
  - the AP® Practice Exams after Chapter 9 to help you prepare for the AP® Exam.

In calculus, reading the text is an essential part of the learning curve and will, in the end, save you time in understanding and mastering the material. There is more to the study of calculus than just being able to do some mathematics; you must understand the concepts and how they fit together. You will also learn to broaden your thinking as well as think logically if you allow yourself to try to see mathematics in a new way, not just as a set of algorithms.

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How to get the most out of your Calculus class

- Know your advanced algebra skills:
  - Linear equations
  - Quadratics (factoring)
  - Functions (parent, transformations, piecewise, odd, even, domain, range)
  - Polynomials (zeros, end behavior)
  - Exponential and logarithmic curves
  - Rational and radical equations
  - Direct, inverse relations
  - Conics (for BC)
- Know your trigonometry:
  - Unit circle (0, 30, 45, 60, and 90 degrees and equivalent radian measures)
  - Symmetry around the unit circle
  - Basic identities
- Know basic sequences and series and when to apply which sequence or series formula.
- Have some knowledge of vectors (BC only).
- Know your calculator:
  - Four functions you should be able to do for the AP® test:
    - Plot the graph of a function in an appropriate window.
    - Find the zeros of a function (solving the equations numerically).
    - Calculate the derivative of a function numerically.
    - Calculate the definite integral numerically.
  - Know how to calculate the value of a function at a specific x: on the graph, 2nd calc, value; on the home screen, $y1$(value).
  - Graph your functions and analyze them as a comparison to the algebra you do. Note: You will not receive credit for a student-drawn graph. This step is to help you develop understanding of the function.

Remember that you have to analyze what the calculator gives you (e.g., how to know when a calculator has a "hole": try $1/(x - 2)$ and look for the value at $x = 2$).

Setting up a review schedule

The AP® Calculus courses are concerned with developing a student's understanding of the concepts while providing experiences with its methods and applications. Both the AB and BC courses require a depth of understanding. If you have been doing your homework steadily and keeping up with the course work, you are in good shape. Organize your notes, homework, and handouts from class by topic. For example, have a set of notes on pre-calculus topics (no longer tested on the AP® exam but essential to your success with calculus), limits, derivative rules and applications, integral rules and applications, series and sequences techniques and methods, and major theorems. If you can summarize the main information on a few pages, by topic, you will find reviewing much easier. Refer to these materials and this study guide as you begin to prepare for the exam. Use your textbook to get more detail as needed.

You will be much more comfortable going into the test if you understand how the test questions are designed and how best to approach them.

The multiple-choice questions often require deeper thinking than may at first be apparent. The free-response questions require a mastery of numerical, algebraic, and graphical approaches to problem solving, as well as an ability to verbally describe the meaning of the question/solution. You must actively do problems to gain understanding and excel in your performance. Athletes don't perform well just by reading books about their sport or by watching others. They must practice. So you, too, just like an athlete, must practice, practice, and practice if you want to do your best!

AP® information before the examination

In February

- Make sure that you are **registered** to take the test. Some schools take care of the paperwork and handle the fees for their AP® students, but check with your teacher or the AP® coordinator to make sure that you are registered. This is especially important if you have a documented disability and need test accommodations. If you are studying AP® independently, or if your school does not have an AP® coordinator, call AP® Services at the College Board at (888) 225-5427 (toll-free in the United States and Canada). You can also email *apstudents@info.collegeboard.org* for the name of the local AP® coordinator, who will help you through the registration process.
- Check on the eligibility of your **calculator.** Go online to

  https://apstudents.collegeboard.org/courses/ap-calculus-ab/calculator-policies

  early enough so that if you need a different calculator, you will have time to get one and to become familiar with it.

By Mid-March

- Begin your review process; set a schedule for yourself that you can follow.

Week before

- Review. Read through your notes. Concentrate on the broad outlines of the course, not the small details. Restudy any concept that you feel needs more attention.
- Begin to gather your materials together for the test.

Night before

- Put all of your **materials** in one place.
- Relax and get a good night's **rest** (this alone could improve your score because you will be able to think more clearly throughout the test).

Things to have on test day

- **Approved graphing calculator** with fresh batteries (you may have a second calculator as a backup, but it must also be a graphing calculator). The calculator must not have a typewriter-style (QWERTY) keyboard, nor can it be a non-graphing scientific calculator or on your phone. (*https://apstudents.collegeboard.org/courses/ap-calculus-ab/calculator-policies*) Calculator memories are **not** cleared for the exam.

> **Insight**
> Be sure your calculator is set in **radian mode** (pi radians, or approximately three radians, is half the circle; 3 degrees is an angle just barely above the x-axis).

- #2 pencils (at least 2) with good erasers.
- A watch (to monitor your pace, but turn off the alarm if it has one).
- A bottle of water and a snack (fruit or power bar).
- Social Security number (if you choose to include it on the forms).
- The College Board school code.
- Photo identification and the admissions ticket.
- Comfortable clothes and a sweatshirt or sweater in case the room is cold.

Schools may have your admissions ticket at the testing site; a photo identification may not be needed at your own school, but check with your AP® coordinator prior to test day.

On the day of the examination, it is wise to eat a good breakfast. Studies show that students who eat a hot breakfast before testing get higher scores. Breakfast can give you the energy you need to power you through the test and more. You will spend some time waiting while everyone is seated in the right room for the right test. That's before the test has even begun. With a short break between Section I and Section II, the AP® Calculus exam can last almost four hours.

Now go get a 5!

Taking the AP® Calculus Examination

To do well on the AP® Calculus examination,

- A student should understand and be able to work with the connections between the graphical, numerical, analytical, and verbal representations of functions.
- A student should be able to use derivatives to solve a variety of problems and understand the meaning of a derivative in terms of rate of change and local linearity.
- A student should use integrals to solve a variety of problems and should understand the meaning of a definite integral in terms of the limit of Riemann sums as well as the net accumulation of change.
- A student also needs to:
    - Understand both parts of the Fundamental Theorem of Calculus.
    - Communicate mathematics in written sentences.
    - Appropriately model a physical situation.
    - Use technology correctly and efficiently.
    - Determine the reasonableness of solutions and understand them in terms of units of measurement, size, and so on.

It is important to realize that a student who is in AP® Calculus is expected to have studied **all** of the prerequisite material. A student should have a mastery of functions and their properties and an understanding of algebra, graphs, and the language of domain, range, symmetry, periodicity, and so on. The student should also understand trigonometry and have a mastery of the basic values in the unit circle and the basic trigonometric identities.

Exam Format

The AP® Calculus examination currently consists of two major sections and each of those has two parts. All sections test proficiency on a variety of topics.

Multiple Choice: Section I has two sets of multiple-choice questions. Part A has 30 questions with an allotted time of 60 minutes and does not allow the use of a calculator. Part B has 15 questions and has 45 minutes allotted to it; this set contains some questions for which a graphing calculator would be needed to answer the questions. The multiple-choice section score is based on the number of questions answered correctly; no points will be deducted for incorrect answers and no points are awarded for unanswered questions.

Free Response: Section II has six free-response questions, and it is broken into two portions. Part A consists of two problems; some parts of some problems may require a graphing calculator and you will be allowed 30 minutes. Part B has four problems and you will be allowed 60 minutes; a calculator is not permitted during this time. Although you may continue working on Part A problems during this 60-minute session, you may no longer use a calculator. Thus, when working on Part A, you must be sure to answer the questions requiring a calculator during that first 30-minute period.

The grade for the examination is equally weighted between the multiple-choice and free-response sections of the exam. You can possibly earn a 5 on the exam even if you miss an entire free-response question. Students taking the BC exam will also receive an AB sub-score grade.

The free-response questions and solutions are published annually after the AP® Reading is completed and can be found at apcentral.collegeboard.com.

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General AP® Test-Taking Strategies

Strategize the test question. Begin somewhere. Ask "What do I need?" and then "How do I get there?" Start with a clear definition; for a question about continuity, you need to have a clear definition of continuity to answer the question fully. (For example, see Question 6 of the 2003 AB Exam.)

- Know what the required tools on your calculator are and know how to access and use them:
  - Plot the graph of a function within a viewing window.
  - Find the zeros of a function (numerically solve equations).
  - Numerically calculate the derivative of a function.
  - Numerically calculate the value of a definite integral.
- Know the relationships between f, f', and f''.
- Know your differentiation and integration rules.
- Underline key components of the questions.
- Reread the question after you have answered it to be sure that you answered the question asked, that you haven't gone too far, or that you haven't contradicted yourself.
- Treat units carefully.
- Set the calculator to THREE decimal places and properly use the store key for intermediate steps (if you round too soon, your final answer will not be correct to the requisite three decimal places). If you choose to write your answer with more than three decimal places, only the first three places are read as your answer.

Strategies for the Multiple-Choice Section

Read the question carefully: Pressured for time, many students make the mistake of reading the questions too quickly or merely skimming them. By reading a question carefully, you may already have some idea about the correct answer. Careful reading is especially important in EXCEPT questions. After you solve the problem and have a solution, reread the question to be sure the answer you solved for actually answers the question. For example, you may have solved for where the maximum occurred (the x-value) but the question actually asks for the maximum value of f (the y-value), and thus you need one more step to complete the problem.

Eliminate any answer you know is wrong: You can write on the multiple-choice questions in the test book. As you read through the responses, draw a line through any answer you know is wrong. Do as much scratch work as is necessary in the exam book, but be sure to mark your solution choice on the answer sheet in the corresponding oval. In most math questions, it is generally better to NOT read through the possible answers until you have found what you think is the solution. There are times where you may have to look at the choices so that you have an idea of how to start or where you are going but generally they can lead you into incorrect assumptions.

Read all of the possible answers, then choose the most accurate response: AP® examinations are written to test your precise knowledge of a subject. Some of the responses may be partially correct, but there will only be one response that is completely true. *Be careful of absolute responses*. These answers often include the words *always* or *never*. They could be correct, but you should try to think of counterexamples to disprove them.

Skip tough questions: Skip them in the first go-through but be sure that you mark them in the margin so you can come back to them later if you possibly can. *Make sure you skip those questions on your answer sheet, too*.

There is no penalty for guessing: Thus, at the end, try to narrow down your choices by eliminating answers which you figure are not correct and make an educated guess. It is to your advantage to answer every question.

Additional Thoughts

- The exact numerical answer may not be among the choices given. You will have to choose the solution that best approximates the exact value.
- The domain of a function f is assumed to be the set of all real numbers x, where $f(x)$ is a real number, unless specified otherwise.
- f^{-1} or the prefix *arc-* indicates the inverse of a trigonometric function (e.g., $\cos^{-1} x = \arccos x$).

Types of Multiple-Choice Questions

All kinds of topics will be covered in the multiple-choice section; your skills and vocabulary will be tested as well as your ability to do multi-step problem solving. Terms like *average value, the definition of continuity, extremum (relative and absolute), the definition of a derivative in its two forms, differential equations, graphical interpretations,* and *slope fields* are just a sampling of terms that identify the kinds of problems you will see. Read through this text and do the practice problems to familiarize yourself with the way the questions are framed.

Multiple-choice questions will be formatted in two basic ways. You will find classic questions where there are just four choices for solutions. This is the most common type of problem; it requires you to read the question and select the most correct answer. Strategies for solving this type of problem include

- reading the question carefully,
- solving the problem and then interpreting your solution correctly to fit the question,
- eliminating known wrong answers, and
- on occasion, testing each solution to see which one is correct.

There will also be problems that could be called "list" and "group," where you may be asked "Which of the following is true about g?" They will give choices such as I, II, and III and the multiple-choice answers might appear as

(A) None
(B) I only
(C) I and II only
(D) I, II, and III

This kind of problem requires a clear understanding of some concept or definition. To approach this kind of question,

- eliminate known wrong answers,
- recall necessary theorems or definitions to help you interpret the question, and
- reread the problem to check your solution's accuracy.

Strategies for the Free-Response Section

- ALL work needs to be shown IN the test booklet.
- Scan all of the questions in the section you are working in. First solve the problems that you think you can do easily. You can mark and come back to the harder ones later. Most questions have multiple entry points. If you cannot answer part a, proceed to the next part, because part a is not always the easiest part.
- Show all of your work. Partial credit will be awarded for problems if the correct work is shown even if the answer is not present or is incorrect. Although not required, it can be helpful to the reader if you circle your final answer.
- Cross out incorrect answers with an "X" rather than spending time erasing. Crossed out or erased work will not be graded. However, don't cross out or erase work unless you have replaced it. Let the reader see what you tried; it may be worth some points.
- Be clear, neat, and organized in your work. If a reader cannot clearly understand your work, you may not receive full credit.

- Some free-response questions have several parts, such as a, b, c, and d. Attempt to solve each part. Even if your answer to "a" is incorrect, you still may be awarded points for the remaining parts of the question if the work is correct for those parts. Remember, the answers may not depend on an earlier response and that is why it is important to try each part. If you work with your incorrect answer (as long as it is reasonably derived) from a previous part, the reader will read with you in a later part (i.e., the reader will check your numerical answer on the basis of your incorrect input).
- Units are important in your answer. Keeping track of your units throughout calculations and performing unit cancellations, where possible, will help guide you to your answer. Points will be deducted for missing or incorrect units in the answer if the question asked that units be given.
- Don't just write equations or numbers in hopes of finding the correct answer. Extraneous or incorrect information could lead to a lower score. Don't make up work that is trivial, but do try the problem and the reader will read with you.
- You do not need to work the questions in order, but be sure the answer is entered in the correct section.
- When you use a table or a graph from one section (part a) in another part of the problem (part c, for example) be sure to refer to it in some way—state your use of it or draw an arrow back to it. If you inadvertently put a response in the wrong part of the problem, again, note it clearly to the reader.
- Show all your work.
  - Clearly label any functions [if the problem uses $g(x)$, don't call it $f(x)$]. If $g(x) =$ to some expression, use $g(x)$ in any problem in lieu of writing the expression. This helps to avoid a copy error.
  - Label your sign charts accurately, for example, f' or f'' for the derivative tests. However, these by themselves do not count as a justification. No credit is given for sign charts, but they can help develop your analysis and subsequent answer to the question. Do not use the pronoun "it" in your descriptions or solutions, specify which function or derivative or variable, etc. that you are using or the part of the question you are answering.
  - Label all graphs with appropriate notation including numeric intervals (by 1s or 10s, for example) and the names for the x- and y-axes (like distance and time).
  - Label all tables or other objects that you use to show your work.
  - Show standard mathematic (non-calculator) notation. For example, you must show the integral as $\int_1^3 (x + 2)\,dx$, not as fnInt $(x + 2, x, 1, 3)$.

Remember: You are not required to simplify your answer. It is best if you leave an answer in an un-simplified form to avoid making careless errors and to save time. For example, $y - 2.3 = -6(x + 5.4)$ would be an appropriate equation of a tangent line; there is no need to simplify it to slope-intercept form.

- Decimals require an accuracy of three decimal places in the solution. Thus be sure to understand how to carry (store in your calculator) the intermediate steps of a problem until you round to three decimal places at the end of the problem. If you do multiple calculations and each calculation is rounded to three decimal places prior to the next calculation, your final solution will not have the required accuracy. The third digit in the final solution can be rounded or truncated.

Scoring for Free-Response Questions

The free-response sections are graded on a scale of 0–9 with a dash (–) given for no work on the page. The chief reader is ultimately responsible for not only working through the solution and alternate solutions for each problem but is also responsible for assigning points on a 9-point scale to each problem. This varies from problem to problem based on how many parts are in the problem, as well as the difficulty or complexity of the particular question.

For example, in a problem that asks for units, units are generally assigned 1 point for the whole problem. In other words, if you do units correctly in part a, but incorrectly in part c, you would not be awarded the 1 point for units.

If a problem requires an explanation, or reasoning, it generally earns 1–2 points. In many cases, explanations can be in writing or with mathematical symbols.

In a typical area and volume problem, the integral is often worth 1 point and the answer is worth an additional 1 point. Sometimes the limits of integration are also worth 1 point. Thus, it is important that you at least start working on a problem because often some points can be earned for the setup, even if the solution is not there or is incorrect.

A "bald" answer is seldom awarded a point. A bald answer is one that has no supporting work or documentation, like "yes" or just a number.

To learn calculus and best prepare for the examination, read the text, take risks, ask questions, and look for the connections between the algebraic, numerical, and graphical approaches to similar problems. As much as possible, graph every problem to enhance your understanding of the concepts you are working with.

P Preparation for Calculus

CalcView®

CalcChat®

Chapter P | Section 1 | Exercise 43 | GO

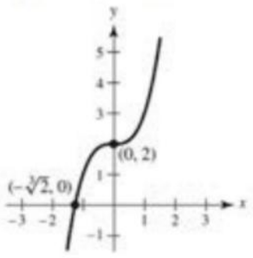
$y = x^3 + 2$

$y = 0^3 + 2 = 2$, y-intercept

$0 = x^3 + 2 \Rightarrow x^3 = -2 \Rightarrow x = -\sqrt[3]{2}$, x-intercept

Intercepts: $(-\sqrt[3]{2}, 0)$, $(0, 2)$

Symmetry: none



PREV. 39 41 43 45 47 49 51 53 55 57 59 NEXT

P.1 Gross Domestic Product *(Exercise 69, p. 11)*

P.3 Automobile Aerodynamics *(Exercise 82, p. 33)*

left, Art Wager/E+/Getty Images; right, Bill Pugliano/Getty Images News/Getty Images

P.1 Graphs and Models

- Sketch the graph of an equation.
- Find the intercepts of a graph.
- Test a graph for symmetry with respect to an axis and the origin.
- Find the points of intersection of two graphs.
- Fit a mathematical model to a real-life data set.

The Graph of an Equation

Your study of calculus will be from multiple perspectives—*graphically*, *analytically*, *numerically*, and *verbally*. By using multiple perspectives, you will increase your understanding of core concepts.

For example, consider the equation $3x + y = 7$. The point $(2, 1)$ is a **solution point** of the equation because the equation is satisfied (is true) when 2 is substituted for x and 1 is substituted for y. This equation has many other solutions, such as $(1, 4)$ and $(0, 7)$. To find other solutions systematically, solve the original equation for y.

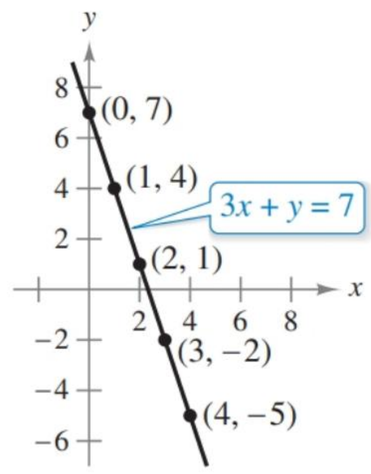
$y = 7 - 3x$ Analytic approach

Then construct a **table of values** by substituting several values of x.

x	0	1	2	3	4
y	7	4	1	-2	-5

Numerical approach

From the table, you can see that $(0, 7)$, $(1, 4)$, $(2, 1)$, $(3, -2)$, and $(4, -5)$ are solutions of the original equation $3x + y = 7$. Like many equations, this equation has an infinite number of solutions. The set of all solution points is the **graph** of the equation, as shown in Figure P.1. Note that the sketch shown in Figure P.1 is referred to as the graph of $3x + y = 7$, even though it really represents only a *portion* of the graph. The entire graph would extend beyond the page. Using a verbal approach, you can say the graph of $3x + y = 7$ is a line that falls from left to right.



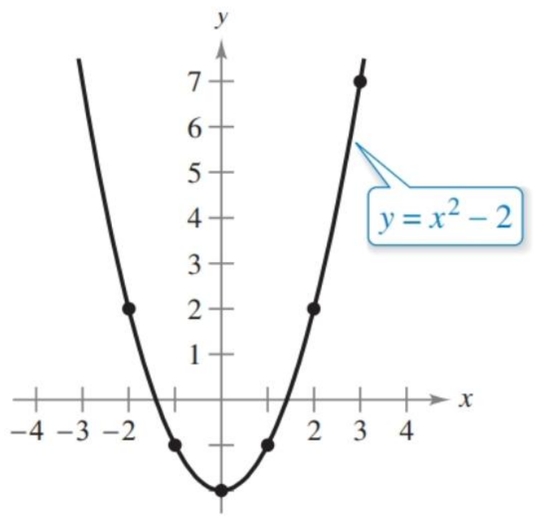
Graphical approach: $3x + y = 7$
Figure P.1

In this course, you will study many sketching techniques. The simplest is point plotting—that is, you plot points until the basic shape of the graph seems apparent.

EXAMPLE 1 Sketching a Graph by Point Plotting

To sketch the graph of $y = x^2 - 2$, first construct a table of values. Next, plot the points shown in the table. Then connect the points with a smooth curve, as shown in the figure below. This graph is a *parabola*. (See Section 9.1 for more about parabolas and other conics.) Verbally, you can say the graph of $y = x^2 - 2$ is a curve that falls from left to right until it reaches its vertex at $(0, -2)$, then it rises from left to right.

x	-2	-1	0	1	2	3
y	2	-1	-2	-1	2	7



The parabola $y = x^2 - 2$

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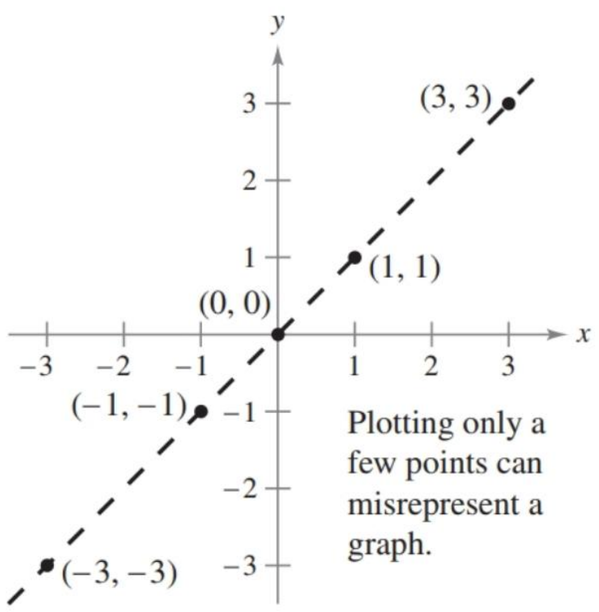
One disadvantage of point plotting is that to get a good idea about the shape of a graph, you may need to plot many points. With only a few points, you could badly misrepresent the graph. For instance, to sketch the graph of

$$y = \frac{1}{30}x(39 - 10x^2 + x^4)$$

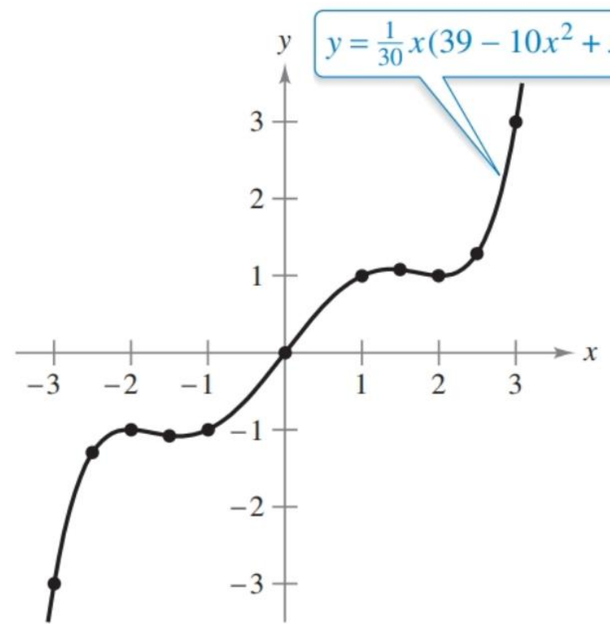
you plot five points:

$$(-3, -3), \quad (-1, -1), \quad (0, 0), \quad (1, 1), \quad \text{and} \quad (3, 3)$$

as shown in Figure P.2(a). From these five points, you might conclude that the graph is a line. This, however, is not correct. By plotting several more points, you can see that the graph is more complicated, as shown in Figure P.2(b).



(a)



(b)

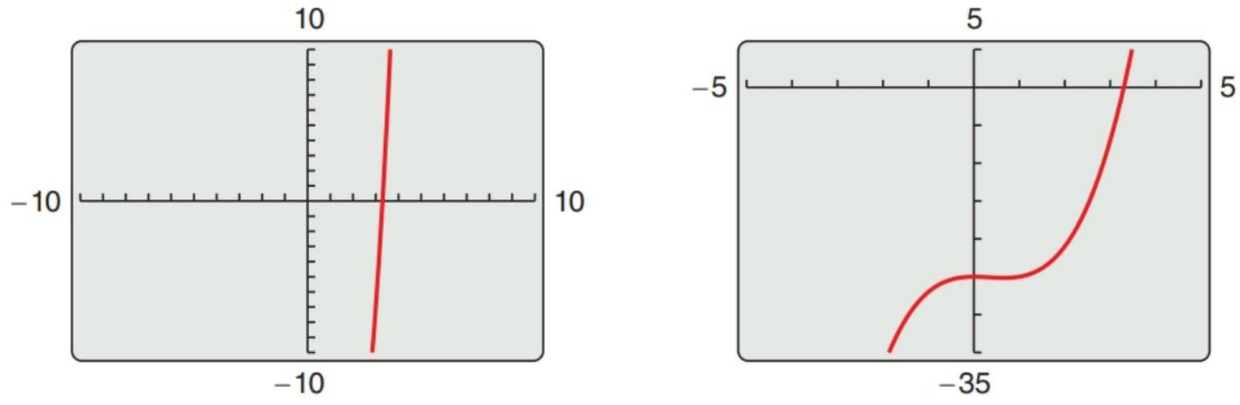
Figure P.2

Technology

Graphing an equation has been made easier by technology. Even with technology, however, it is possible to misrepresent a graph badly. For instance, each of the graphing utility* screens in Figure P.3 shows a portion of the graph of

$$y = x^3 - x^2 - 25.$$

From the screen on the left, you might assume that the graph is a line. From the screen on the right, however, you can see that the graph is not a line. So, whether you are sketching a graph by hand or using a graphing utility, you must realize that different "viewing windows" can produce very different views of a graph. In choosing a viewing window, your goal is to show a view of the graph that fits well in the context of the problem.



Graphing utility screens of $y = x^3 - x^2 - 25$

Figure P.3

Exploration

Comparing Graphical and Analytic Approaches Use a graphing utility to graph each equation. In each case, find a viewing window that shows the important characteristics of the graph.

a. $y = x^3 - 3x^2 + 2x + 5$
b. $y = x^3 - 3x^2 + 2x + 25$
c. $y = -x^3 - 3x^2 + 20x + 5$
d. $y = 3x^3 - 40x^2 + 50x - 45$
e. $y = -(x + 12)^3$
f. $y = (x - 2)(x - 4)(x - 6)$

A purely graphical approach to this problem would involve a simple "guess, check, and revise" strategy. What types of things do you think an analytic approach might involve? For instance, does the graph have symmetry? Does the graph have turns? If so, where are they? As you proceed through Chapters 1, 2, and 3 of this text, you will study many new analytic tools that will help you analyze graphs of equations such as these.

*In this text, the term *graphing utility* refers to graphing calculators (such as the *TI-84 Plus* and *Desmos*) and computer graphing software (such as *Maple* and *Mathematica*).